Tobias Jahnke

Universität Karlsruhe (TH), Department of Mathematics
Institute for Applied and Numerical Mathematics
jahnke@math.uni-karlsruhe.de

An adaptive wavelet method for chemical master equations

Many biological processes are modeled as complex reaction systems in which different species interact via a number of reaction channels. Often the evolution of the entire system is crucially determined by one or two sub-populations which may consist of a very small number of individuals. In this situation, small changes in the critical sub-population due to inherent stochastic noise can cause large-scale effects. Hence, a reasonable mathematical model of such processes must respect both the stochasticity of the time evolution and the discreteness of population numbers.

This insight has fostered an ever increasing interest in stochastic reaction kinetics. Here, the system is described by a time-dependent probability distribution $p(t,x)$ which, for each state $x = (x_1, \ldots, x_d) \in \mathbb{N}^d$, indicates the probability that at time t exactly x_i individuals of the $i-$th species exist. The probability distribution evolves according to the chemical master equation, which is a particular type of the forward Kolmogorov equation. Unfortunately, solving the chemical master equation is often a formidable task due to the tremendous number of degrees of freedom, which originates from the fact that the solution $p(t,x)$ has to be approximated in each state x of a large and multi-dimensional state space.

In this talk, we present an adaptive method which allows to reduce the large number of degrees of freedom considerably. The method is based on a spatial representation of the probability distribution in a sparse Haar wavelet basis, time discretisation in the framework of the Rothe method, and a strategy which in each time-step selects the basis vectors required to approximate the solution up to the desired accuracy. The accuracy of the method is discussed, and its efficiency is illustrated by a numerical example.

The talk is based on a joint work with Steffen Galan (Freie Universität Berlin/Germany).